3.18

$$a)\nabla^2 V = \nabla^2(xy^2z^3) = 0 + 2xz^3 + 6xy^2 z$$

$$b)\nabla^2(xy + yz + zx) = 0$$

$$c)\nabla^2(\frac{1}{x^2+y^2}) = \nabla^2(\frac{1}{r^2}) = \frac{1}{r}\frac{\partial}{\partial r}[(r\frac{\partial}{\partial r}(\frac{1}{r^2})]$$

$$= \frac{4}{r^4} = \frac{4}{(x^2+y^2)^2}$$

$$d)\nabla^2(5e^{-r}\cos\phi) = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}V) + \frac{1}{r^2}\frac{\partial^2 V}{\partial\phi^2}$$

$$= 5\cos\phi\frac{1}{r}\frac{\partial}{\partial r}(-re^{-r}) - \frac{5e^{-r}\cos\phi}{r^2}$$

$$= 5\cos\phi[\frac{(-e^{-r} + re^{-r})}{r} - \frac{e^{-r}}{r^2}]$$

$$= 5e^{-r}\cos\phi(1 - \frac{1}{r} - \frac{1}{r^2})$$

$$e)\nabla^2(10e^{-R}\sin\phi) = \frac{1}{R}\frac{\partial}{\partial R}(R^2\frac{\partial V}{\partial R}) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial}{\partial\theta})$$

$$= \frac{1}{R}\frac{\partial}{\partial R}(-10R^2e^{-R}\sin\theta) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial\theta}(10e^{-R}\sin\theta\cos\theta)$$

$$= \frac{1}{R^2}[-20R(e^{-R})\sin\theta + 10R^2e^{-R}\sin\theta] + \frac{1}{R^2\sin\theta}10e^{-R}[\cos^2\theta - \sin^2\theta]$$

$$= 10e^{-R}[\sin\theta(1 - \frac{2}{R}) + \frac{\cos^2\theta - \sin^2\theta}{R^2\sin\theta}]$$