Lin *et al.*[9] in which the dark lines are interpreted as solitons.

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Soliton Propagation in Liquid Crystals

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Soliton propagation in nematic liquid crystals under shear is shown to be possible and studied theoretically. Calculations including those pertaining to the modulation of monochromatic or white light passing through such a liquid-crystal cell are presented. Recent experiments are interpreted accordingly and are in good agreement with the theory presented here.

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Solitons are important and have been found in various objects ranging from celestial bodies to laboratory systems.[1,2] However, unlike the first observation of solitons in shallow water by Scott Russell, many of the recent experimental evidences of solitons in condensed matter are indirect in nature. The experiments[3] on the ordered fluid ^{3}He are no exception. In this regard, we note that in another type of ordered fluid, viz., liquid crystal, because of the strong coupling of the director with light, it may be possible to observe the motion of the molecules and the solitons rather directly.

Discussions of solitons in liquid crystals[4] was first given by Helfrich[5] and subsequently by de Gennes,[6] Brochard,[6] and Leger.[7] In their work in nematics, the solitons (called "walls") are magnetically generated and are small in width (e.g., a few microns). Experimentally, the observation[7] of these solitons is delicate and a polarizing microscope has to be used. Recently, there has been more but still limited attention[8] paid to the role of solitons in the physics of liquid crystals.

In this Letter, we first point out and discuss a new case in liquid crystals, viz., nematics under uniform shear, in which solitons can exist and propagate. In contrast to the magnetic case[5-7] the propagation of these solitons can be observed even by the naked eye and easily measured. We then give an analysis and explanations of some recent experiments[9,10] which are found to be in good agreement with our theory.

Let us consider a nematic under uniform shear such that the velocity is given by $\vec{v}=(v(y),0,0)$ and $s\equiv\partial v/\partial y=$ const. The incompressibility condition $\nabla\cdot\vec{v}=0$ is clearly satisfied. Under the assumptions that the director $\vec{n}=(\sin\theta,\ \cos\theta,\ 0)$ and $\theta=\theta(x,t)$ we find, according to the Ericksen-Leslie equations, that[11]

$$M\frac{d^2\theta}{dt^2}=K\frac{\partial^2\theta}{\partial x^2}-\gamma_1\frac{d\theta}{dt}+\frac{s}{2}(\gamma_1-\gamma_2\cos2\theta), \quad (1)$$

where the one-constant assumption ($K_1=K_2=K_3=K$) is used. Here, M is the moment of inertia, K the elastic constant, γ_1 and γ_2 the viscosity coefficients, and $d\theta/dt=\partial\theta/\partial t+v\partial\theta/\partial x\simeq\partial\theta/\partial t$ is assumed. Equation (1) is the damped driven sine-Gordon equation which is known to have soliton solutions.[12] When θ is a traveling wave of velocity c, Eq. (1) becomes

$$m\ddot{\theta}=-\eta\dot{\theta}-\partial U/\partial\theta, \quad (2)$$

where $\theta=\theta(Z)$, $Z\equiv X-\eta T$, $X\equiv x/\lambda$, $T\equiv t/\tau$, $\lambda\equiv(2K/|\gamma_2|s)^{1/2}$, $\tau\equiv2\gamma/s$, $\eta\equiv c\tau/\lambda$, $m=1-Ms\eta^2/(2\gamma^2|\gamma_2|)$, $\gamma\equiv\gamma_1/|\gamma_2|$, $U=\gamma\theta+\frac{1}{2}\sin2\theta$, and $\dot{\theta}\equiv d\theta/dZ$. In (2), the experimental fact that $\gamma_2<0$ is adopted and $s>0$ is assumed. Note that if θ is the soliton for $s>0$ then $-\theta$ is that for $s<0$.

Equation (2) describes the damped motion of a particle with mass m in an apparent potential U. The damping coefficient is η and Z plays the role of time. For $0<\gamma<1$, U has a series of maxima at $\theta=\theta_0+k\pi$ and minima at $\theta=-\theta_0+k\pi$ where k is an arbitrary integer and $\pi/4<\theta_0\equiv\frac{1}{2}\cos^{-1}(-\gamma)<\pi/2$. There are only three types of solitons corresponding respectively to the particle starting (with zero velocity) at the maximum at $\theta=\theta_0$ and ending in (A) the adjacent minimum at $\theta=-\theta_0$, (B) the minimum at $\theta=\pi-\theta_0$, or (C) the maximum at $\theta=\theta_0-\pi$. Type C appears only when $\eta=\eta_c$ but type A (B) is possible for all $\eta>\eta_c$ ($\eta>0$). Here, η_c is a parameter which increases monotonically with γ from zero at $\gamma=0$ to $0.84m^{1/2}$ at $\gamma=1$ (see Fig. 7 of Ref. 12). Note that there is no soliton for $\gamma>1$. For $\gamma=1$, type A reduces to type C. With the experiment of Ref. 10 in mind, we will discuss below only the strongly damped case of $\eta\gg1$.

To this end, (2) is expanded in $1/\eta$ resulting in

$$\dot{\theta}=-(\gamma+\cos2\theta)/\eta-2(\sin2\theta)\dot{\theta}/\eta^2+O(1/\eta^3) \quad (3)$$

which has been solved analytically.[13] Numerically, soliton solutions of type A may be approximated very accurately by the more simple expression

$$\theta=\tan^{-1}\{w\tanh[(\gamma-1)wZ/\eta]\}, \quad (4)$$

where $w\equiv[(1+\gamma)/(1-\gamma)]^{1/2}$, which is actually the solution of (3) to $O(1/\eta)$. As expected, θ decreases monotonically from θ_0 at $Z=-\infty$ to $-\theta_0$ at $Z=+\infty$ ($\gamma<1$), the two uniform states allowed by the shear flow. In (3) and (4), without loss of generality, $m=1$ is assumed.

When the shearing nematic is placed between two crossed polarizers which are in the x-z plane with the polarizing direction at 45° with the x axis, the part of the soliton corresponding to $\theta=0$ will appear as a dark line moving with velocity c in the x direction. The illuminating light is assumed to be in the y direction.

For monochromatic light of wavelength λ_0 the ratio of output to input intensities I/I_0 as a function of Z/η is calculated. It varies from 0 to 1 consisting of a series of minima and maxima. The positions of the points with $I/I_0=0.5$ are depicted in Fig. 1. The region between two adjacent points with a minimum in between is painted black. In Fig. 2, the curve I/I_0 for white light is shown. The width of the dark line at the center, Δ, is found to decrease with γ as shown in Fig. 3. In these calculations, (4) is used; thickness of the nematic $2d=20\ \mu$m, refractive indexes $n_0=1.54$ and n_e from Fig. 6 of Ref. 14 for N-[p-methoxybenzylidene]-p-butylaniline (MBBA) are adopted.

What we discussed above is the appearance of the solitons once they are created. There remains the question of how the solitons can be excited. In the experiments of Ref. 10, nematic

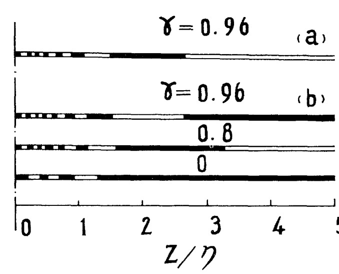

FIG. 1. Theoretical "photograph" of transmitted monochromatic light derived from the calculated I/I_0 vs Z/η curve (see text). The picture is symmetric in Z and $-Z$. (a) $\lambda_0=6328$ Å, $\gamma=0.96$; (b) $\lambda_0=6000$ Å, $\gamma=0.96$, 0.8, 0.

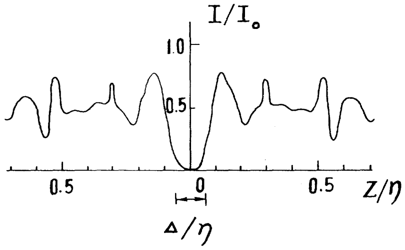

FIG. 2. Transmitted white light intensity I/I_0 vs Z/η. $\gamma = 0.96$. Δ is the width of the dark line defined at the half maximum intensity.

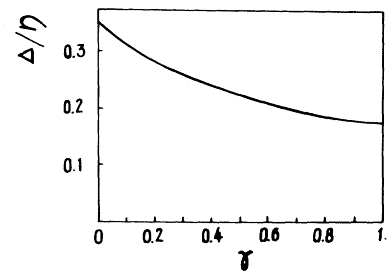

FIG. 3. Dependence of the width of the dark line Δ/η on γ.

MBBA is homeotropic and at rest initially. To excite the solitons, a Mylar plate placed at one end of the liquid-crystal cell is either pushed or pulled steadily along the cell. In our opinion, the movement of this plate creates velocity gradients in the nematic.[15] To first approximation, the velocity profile may be assumed to be steady and of the form shown in Fig. 4. The problem becomes one dimensional and our results presented above may be applied without modification to each of the layers (of thickness d) near the surfaces of the cell.

The good agreement between our theory and the experimental results[10] are evidenced by the following. (i) The behavior of the dark lines in the two cases of pushing and pulling of the plate are similar to each other.[10] Our theory predicts identical behavior when all other conditions are identical. (ii) Experimentally, $c \gg v$. Our theory gives $\delta \equiv \lambda\Delta = (c/s)2\gamma f(\gamma)$ where $f(\gamma)$ is the curve in Fig. 3, implying that a thick dark line (under white light) moves faster than a thin one, in agreement (at least qualitatively) with experiments.[10] (iii) Experiment and our theory both show that the dark line corresponds to molecules normal to the glass plates. (iv) The characteristics of I/I_0 shown in our Fig. 1 are in agreement with that in Ref. 10. In fact, the experimental pattern of the transmitted monochromatic light may be understood as resulting from the overlapping of three patterns of the type similar to Fig. 1 (corresponding to three solitons) as evidenced from theoretical results[13] shown in Fig. 5. (v) For a cell of 30 cm in length, 5 cm in width, and $d = 10$ μm, $K = 10^{-6}$ dyn, $\gamma = 0.96$, $c = 10$ cm/sec, and $V = 0.05$ cm/sec (resulting in $\theta_0 = 81.9°$, $\eta = 1.6\times10^3$), the power required to generate and maintain the propagation of one soliton is calculated to be ~ 193 erg/sec. The experimental result (for three solitons)[16] is $\sim 10^2$ erg/sec. With the same set of parameters and from our Fig. 3 we find $\delta = 0.8$ mm while experiment gives $\delta \sim 1$ mm. Note that physically V is always smaller than the velocity of the pushing plate. (vi) The dark line (under white light) is sandwiched between two bright narrow lines (see Fig. 2). This is clearly observed experimentally.[10]

Knowing the temperature dependence[17] of γ, n_0, and n_e, one may obtain δ as a function of temperature. Also, using $s = V/d$ our theory predicts $\delta/c = 2\gamma f(\gamma)d/V$. This can be checked easily by varying the thickness of the cell or of the pushing plate. The occurrence of three dark lines in the experiment[10] is related to the input power and the

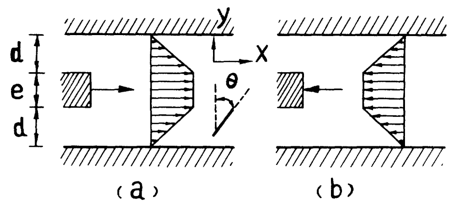

FIG. 4. Velocity profiles created by the pushing (a) or pulling (b) of the Mylar plate at the left of the cell. The maximum velocity in the profile is V.

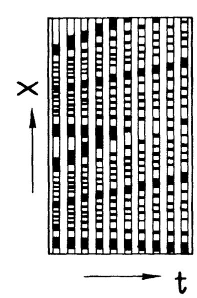

FIG. 5. Modulated monochromatic light pattern corresponding to two solitons (see Ref. 13). $\lambda_0 = 6328$ Å, $\gamma = 0.96$.

shear rate. Only one dark line is observed when a thinner pushing plate is used,[16] or when the soliton is generated by applying pressure gradients.[2,18]

Strictly speaking, those theoretical results obtained above for a traveling wave are applicable only in the time region in which the velocity of the dark lines is almost constant.[13] The major features and conclusions of the above one-dimensional analysis are retained in a more refined two-dimensional study.[13]

Solitons of type B should be observable for nematics initially in planar configuration.[13] With the setup described in Ref. 10 dark lines (under white light) cannot be (and have not been[16]) observed when the homeotropic configuration is replaced by the planar one. For both homeotropic and planar configurations, type C solitons should be observable. Note that the method used in Ref. 10 to excite solitons is not unique. Further discussions and other results can be found in Refs. 2 and 13.

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